When $x=0$, (1) becomes

$$\frac{1}{1.2.2.3}-\frac{1}{2.3.3.4}+\frac{1}{3.4.4.5}-\frac{1}{4.5.5.6}+\ldots=\frac{\pi^2}{12}-\frac{3}{4}\ldots(3).$$

When $x=\pi$, (1) becomes

$$\frac{1}{1.2.2.3}+\frac{1}{2.3.3.4}+\frac{1}{3.4.4.5}+\frac{1}{4.5.5.6}+\ldots=\frac{7}{4}-\frac{\pi^2}{6}\ldots(4).$$

Subtracting (3) from (4), we find the sum of the given series to be $\frac{5}{4}-\pi^2/8$.

Also solved by J. Scheffer.

315. Proposed by PROFESSOR B. F. YANNEY, Mount Union College, Alliance, Ohio.

Simplify, $1-(2-(3-\ldots-(n-1)-n)\ldots)))$.

Solution by GEORGE W. HARTWELL, University of Kansas, Lawrence, Kansas, and V. M. SPUNAR, Pittsburg, Pa.

Removing the parentheses, this expression becomes

$$1-2+3-4+\ldots(-1)^{n-1}n\equiv\sum_1^n(-1)^{n-1}n.$$

But $\sum_1^n(-1)^{n-1}n=-(n/2)$ when n is even,

and $\sum_1^n(-1)^{n-1}n=(n+1)/2$ when n is odd.

Also solved by G. B. M. Zerr.

GEOMETRY.

342. Proposed by G. I. HOPKINS, M. A., Instructor in Mathematics and Astronomy, Manchester, N. H.

Given, circle DEF inscribed in triangle ABC and circumscribing the triangle DEF, D, E, F being the points of contact; AH is drawn through center, N, meeting chord DF in H. Through H is drawn BK meeting AC in K. Prove triangle ABK isosceles.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let the points D, F, E be situated on the sides a, b, c, respectively, and also let $l=\cos^2(A/2)$, $m=\cos^2(B/2)$, $n=\cos^2(C/2)$. Then $(0, rn, rm)$; $(rn, 0, rl)$, are the trilinear coordinates of D and F, respectively.

Hence $\beta-\gamma=0$ is the equation to AN, $l\alpha+m\beta-n\gamma=0$ is the equation to DF.

$\therefore (n-m,\ l,\ l)$; $(0,\ 2\triangle/b,\ 0)$, are the coordinates of H and B, respectively. $\therefore l\alpha+(m-n)\gamma=0$ is the equation to BK.

But $m-n+l\cos C+(m-n)\cos A-l\cos B=0$.

$\therefore BK$ is perpendicular to AN. $\therefore$ triangle ABK is isosceles.

II. Solution by G. I. HOPKINS, Instructor in Mathematics and Astronomy, High School, Manchester, N. H.

Construction: Join HE, HB. Extend DE and draw BP perpendicular to it.

Demonstration: Since BP and AH are perpendicular to DE, they are parallel. $AD=AE$, *i. e.*, $\triangle ADE$ is isosceles. arc ES=arc SF.

$\therefore \angle ENB=\angle EDF$. $\therefore \triangle DRH$ and $\triangle NEB$ are similar.

$\therefore NE/EB=DR/RH$. $\angle NEB$ is a right angle.

$\therefore \angle REN$ is complement to $\angle BEP$. $\therefore \angle REN=\angle EBP$.

$\therefore \triangle REN$ and $\triangle EBP$ are similar.

$\therefore RE/BP=NE/EB$; $\therefore DR/RH=RE/BP$.

Since $DR=RE$, $\therefore RH=BP$, and $\therefore RHBP$ is a parallelogram; *i, e.*, BK is parallel to DE.

$\therefore \triangle ABK$ is similar to $\triangle ADE$, and is therefore isosceles.

CALCULUS.

270. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that $\sum\limits_{x=0}^{x=\infty}\dfrac{1}{(a^2+x^2)^n}=\dfrac{\pi}{2a^{2n-1}}\cdot\dfrac{1}{2}\cdot\dfrac{3}{4}\cdot\dfrac{5}{6}\cdots\dfrac{(2n-3)}{(2n-2)}+\dfrac{1}{2a^{2n}}$, n being a positive integer >1.

II. Solution by the PROPOSER.

Performing the finite summation of the problem by the aid of Maclaurin's Summation-formula,

$$\Sigma\, u_x=C+\int u_x dx-\tfrac{1}{2}u_x+\frac{B_1}{2!}\frac{du_x}{dx}-\frac{B_2}{4!}\frac{d^3\,u_x}{dx^3}+\dots$$

(See Boole's *Finite Differences*, page 90), we readily obtain the above expression for the sum, if we substitute for the definite integral, $\int_0^\infty\dfrac{dx}{(a^2+x^2)^n}$, its well known value, $\dfrac{\pi}{2}\cdot\dfrac{1}{2}\cdot\dfrac{3}{4}\cdot\dfrac{5}{6}\cdots\dfrac{(2n-3)}{(2n-2)}\cdot\dfrac{1}{a^{2n-1}}$, if n is a positive integer >1.

The solution in the May MONTHLY involves the erroneous assumption that $\sum\limits_{x=0}^{x=\infty}\dfrac{1}{(a^2+x^2)^n}=\int_0^\infty\dfrac{dx}{(a^2+x^2)^n}$.